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FORMULATION OF THE SIMPLE MARKOVIAN MODEL USING FRACTIONAL CALCULUS APPROACH AND ITS APPLICATION TO ANALYSIS OF QUEUE BEHAVIOUR OF SEVERE PATIENTS

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ABSTRACT

In this paper, we introduce a fractional order of a simple Markovian model where the arrival rate of the patient is Poisson, *i.e.* independent of the patient size. Fraction is obtained by replacing the first order time derivative in the difference differential equations which govern the probability law of the process with the Mittag-Leffler function. We derive the probability distribution of the number $N(t)$ of patients suffering from severe disease at an arbitrary time t. We also obtain the mean size (number) of the patients suffering from severe disease waiting for service at any given time t, in the form of $E^{\nu}_{0.5,0.5}(t)$, for different fractional values of server activity status, $\nu = 1, 0.95, 0.90$ and for arrival rates $\alpha = \beta = 0.5$. A numerical example is also evaluated and analysed by using the simple Markovian model with the help of simulation techniques.

Key words: fractional order, arrival rate, patients, fractional calculus.

1 Introduction

From the historical point of view, fractional calculus may be described as an extension of the concept of a derivative operator from integer order n to arbitrary order α, where α is a real or complex value, or even more complicated, a complex valued function,

$$\alpha = \alpha(x,t) \tag{1}$$

Despite the fact that this concept has been discussed since the days of Leibniz (1695) and since then has occupied the great mathematicians of their times, no other research area has resisted as much a direct application for centuries. Abel's treatment of the tautochrone problem from 1823 stood for a long time as a singular example of an application for fractional calculus. Abel (1823) define the equation as

$$\frac{d^n}{dx^n} \longrightarrow \frac{d}{dx} \tag{2}$$

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Differentiation and integration are usually regarded as discrete operations, in the sense that we differentiate or integrate a function once, twice, or any whole number of times. But in the case of integer order functions, the question is how to differentiate or integrate the same. Fractional calculus is useful to evaluate the integer order function.

Fractional Calculus is a significant topic in mathematical analysis as a result of its increasing range of applications, that grows out of the traditional definition of the integer order calculus of derivatives and integrals. It provides several tools for solving differential and integral equations of fractional order. In the recent years, fractional calculus has played a very important role in various fields, based on the wide applications in engineering and sciences such as physics, mechanics, chemistry, biology, applied mathematics, probability and statistics etc.

The application-oriented approaches of fractional calculus are given in many textbooks. For examples, Oldham(1974), Samko (1993), Miller (1993), Kiryakova (1994), Rubin (1996), Gorenflo (1997), Podlubny (1998), Hilfer (2000), Hilfer (2008), Mainardi (2010), Herrmann (2014). These books are explicitly devoted to the practical consequences of using fractional calculus.

There have been few studies related to point processes governed by difference-differential equations containing fractional derivative operators. These processes are direct generalizations of the classical $M/M/1$ queue and the linear birth-death processes. It is well known that a fractional derivative operator induces a non-Markovian behaviour into a system as derived by Veillette (2010). Srivastava (2001) studied a systematic (and historical) investigations carried out by various authors in the field of fractional calculus and its applications. Srinivasan (2008) has considered a brief elementary and introductory approach to the theory of fractional calculus and its applications especially in developing solutions of certain families of ordinary and partial fractional differential equations.

Moreover, parameter estimation and path generation algorithms of these new fractional stochastic models were derived. It is to be noted that the proposed fractional point models (with Markovian and non-Markovian properties) are parsimonious, which makes them desirable for modelling real-world non-Markovian queuing systems. Dhar (2014) studied the comparison between single and multiple Markovian queuing model in an outpatient department. Also, Mahanta (2016) proposed a single server queueing model for severe diseases especially in outpatient department. Further, consider the infinite server queues with time-varying arrival and departure pattern when the parameters are varying with time derived by Dhar(2017).

It is further observed that more recently fractional point processes driven by fractional difference-differential equations such as the fractional Poisson, the fractional birth, the fractional death, and the fractional birth-death processes have already been gaining attention as studied by Beghin (2009), Cahoy (2010), Laskin (2003), Orsingher (2011).

Recently, Uchaikin (2008), Orsingher (2010), Cahoy (2013) have developed the generalizations of the classical birth and death processes by using the techniques of fractional calculus. A major advantage of these models over their classical coun-

terparts is that they can capture both Markovian and non-Markovian structures of a growing or decreasing system.
Uchaikin (2008) partially investigated the fractional linear birth process by using the Riemann-Liouville derivative operator but it was generalised by Orsingher (2010) using the Caputo derivative. Cahoy (2013) derived the inter-birth time distribution using simulation method to simulate the ${}_fY_p$.
A situation of fractional calculus may be applicable in queuing system when the server is found not working, either from the start or in between. A classic example may be the absence of doctor(s) or his/her leaving the hospital in between for other works, despite patients waiting for treatment.
To date, no practical implementation for any real-life problem has been attempted using the theories as mentioned above. In this paper, an attempt is made to develop a model using the concept of fractional calculus on a queuing system for emergency service of severe patients.
In certain departments, like outpatient department, of many public hospitals, unavailability of doctors during working hours has become a trend these days. These doctors come to their department only at a time convenient for them. The outpatient department of a hospital is visited by patients of all types of disease. Some of these diseases require immediate medical attention as severe complications may arise if treatment is delayed. This delay is commonly due to server inactivity, which may be total or in fractions. By 'fractions' we imply that some portions of the server is active while some is not. Examples may be like, i) doctor is present whereas registration desk personnel is not, or ii) all personnel are present but there is some technical lapse, or iii) registration desk is in order but doctor is not present, and so on. Patients coming from far-off places, postponing their own schedule and engagements, are thus deprived of timely medical services. A system, therefore, must be put in place to make the irregularity of doctors fall in line, so that there is a check on the server system functioning as doctors of these hospitals.

2 Basic Preliminaries

The basic definitions and properties of the fractional calculus theory used in this study are given below.
Definition 1: Let $y = f(t)$ be a continuous (but not necessarily differentiable) function and let partition $h > 0$ in the interval $[0,1]$. Then, the fractional derivative is defined by Podlubny (1998)

$$D^n(f) = \frac{d^n f}{dt^n} = \lim_{h\to 0} \frac{\sum_{j=0}^{n}(-1)^j \binom{n}{j} f(t-jh)}{h^n}$$

If n is fixed then $D^n f \to 0$ as $h \to 0$.
Definition 2: Grunwald Letnikov differential integral of arbitrary order q is defined

by Podlubny(1998)

$$D_a{}^q f(t) = \lim_{N\to\infty} h_N{}^{-q} \left[\sum_{j=0}^{N} (-1)^j \binom{q}{j} f(t - jh_N) \right]$$

where

$$\begin{aligned} \binom{q}{0} &= 1 \\ \binom{q}{j} &= \frac{q(q-1)\dots(q-j+1)}{j!},\ j \in N \end{aligned}$$

lemma 1:

$$\frac{d^n}{dt^n} D_a{}^q f(t) = D_a{}^{n+q} f(t)$$

Definition 3: The Riemann-Liouville fractional integral operator of order $a > 0$ is defined Mathai (2008) as

$$I_a{}^q f(t) = \frac{1}{\Gamma(q)} \int_a^t (t-\tau)^{q-1} f(\tau) d\tau,\ \ t > a$$

Definition 4: The Riemann-Liouville fractional derivative operator of order a is defined [Haubold (2011)] as

$$D_a{}^q f(t) = \frac{d^n}{dt^n} \left[\frac{1}{\Gamma(n-q)} \int_a^t (t-\tau)^{n-q-1} f(\tau) d\tau \right]$$

2.1 Mittag-Leffler function

The Mittag-Leffler function, which plays a very important role in the fractional differential equations was in fact introduced by Mittag-Leffler in 1903. It is a generalization of the exponential series, *i.e.* if $\alpha = 1$ then we have the exponential series. The Mittag-Leffler function $E_a(t)$ is defined by the power series (3)

$$E_a(t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(an+1)},\ a > 0 \tag{3}$$

which gives the generalized Mittag-Leffler function (1.4) as defined

$$E_{\alpha,\beta}(t) = \sum_{n=0}^{\infty} \frac{t^n}{\Gamma(\alpha n+\beta)},\ \alpha,\beta > 0 \tag{4}$$

This generalization was studied by Saxena (2005) and Haubold (2011).

3 Generation of single server queuing model applying fractional concept

Consider a single-server queue with inter-arrival time and service time which are exponentially distributed with rates λ and μ, respectively. Let $N(t)$ be the number of patients in the system at time t. We define

$$p_n(t) \quad = \quad Pr[N(t) = n|N(0) = i], \quad i \geq 0 \tag{5}$$

$M/M/1$ is a special case of the general birth-and-death model with $\lambda_n = \lambda$ and $\mu_n = \mu$.
The generator matrix is given by (state space: $0, 1, 2, \ldots$)

$$M = \begin{bmatrix} -\lambda & \lambda & \ldots & 0 & \\ \mu & -(\lambda+\mu) & \lambda & \ldots & 0 \\ \vdots & \ddots & \vdots & & \end{bmatrix}$$

Then, the governing differential-difference equations of the system under consideration are given by

$$\begin{cases} \frac{\delta p_0(t)}{\delta t} = -\lambda p_0(t) + \mu p_1(t) \\ \frac{\delta p_n(t)}{\delta t} = -(\lambda+\mu) p_n(t) + \mu p_{n+1}(t) + \lambda p_{n-1}(t), n \geq 1 \end{cases} \tag{6}$$

The generator matrix is given by (state space: $0, 1, 2, \ldots$) the matrix below, using the generalized Mittag-Leffler function.

$$E_{\alpha,\beta}(Mt^{\alpha}) \quad = \quad \sum_{n=0}^{\infty} \frac{(Mt^{\alpha})^n}{\Gamma(\alpha n+\beta)} \begin{bmatrix} -\lambda^n & {C_n}^1\lambda^{n-1} & \ldots & {C_n}^{n-1}\lambda^{n-k-1} & \\ \mu^n & -(\lambda+\mu)^n & {C_n}^1\lambda^{n-1} & \ldots & 0 \\ \vdots & \ddots & \vdots & & \end{bmatrix}$$

$$= \quad \sum_{n=0}^{\infty} \frac{(t^{\alpha})^n}{\Gamma(\alpha n+\beta)} \begin{bmatrix} -\lambda^n & {C_n}^1\lambda^{n-1} & \ldots & {C_n}^{n-1}\lambda^{n-k-1} & \\ \mu^n & -(\lambda+\mu)^n & {C_n}^1\lambda^{n-1} & \ldots & 0 \\ \vdots & \ddots & \vdots & & \end{bmatrix}$$

$$= \begin{bmatrix} -\sum_{n=0}^{\infty} \frac{(t^{\alpha})^n\lambda^n}{\Gamma(\alpha n+\beta)} & \sum_{n=0}^{\infty} \frac{(t^{\alpha})^n {C_n}^1\lambda^{n-1}}{\Gamma(\alpha n+\beta)} & \ldots & \sum_{n=0}^{\infty} \frac{(t^{\alpha})^n {C_n}^{n-1}\lambda^{n-k-1}}{\Gamma(\alpha n+\beta)} \\ \sum_{n=0}^{\infty} \frac{(t^{\alpha})^n(\lambda+\mu)^n}{\Gamma(\alpha n+\beta)}\mu^n & -\sum_{n=0}^{\infty} \frac{(t^{\alpha})^n {C_n}^1\lambda^{n-1}}{\Gamma(\alpha n+\beta)} & \ldots & \\ \vdots & \ddots & \vdots & \end{bmatrix}$$

$$= \begin{bmatrix} -E_{\alpha,\beta}(t^{\alpha}\lambda) & \frac{1}{1!}\frac{d}{d\lambda}E_{\alpha,\beta}(t^{\alpha}\lambda) & \ldots & \frac{1}{(k-1)!}(\frac{d}{d\lambda})^{k-1}E_{\alpha,\beta}(t^{\alpha}\lambda) \\ E_{\alpha,\beta}(t^{\alpha}\mu) & E_{\alpha,\beta}(t^{\alpha}(\lambda+\mu)) & \ldots & \\ \vdots & \ddots & \vdots & \end{bmatrix}$$

Here, we assume to satisfy the difference-differential equations for the state probabilities with arrival rate $\lambda > 0$, service rate $\mu > 0$ and $i \geq 0$ initial patients, and we get,

$$\begin{cases} \frac{\delta^\nu p_0^\nu(t)}{\delta t^\nu} = -E_{\alpha,\beta}(t^\alpha \lambda)p_0^\nu(t) + E_{\alpha,\beta}(t^\alpha \mu)p_1^\nu(t) \\ \frac{\delta^\nu p_n^\nu(t)}{\delta t^\nu} = -E_{\alpha,\beta}(t^\alpha(\lambda+\mu))p_n^\nu(t) + E_{\alpha,\beta}(t^\alpha \mu)p_{n+1}^\nu(t) \end{cases} \tag{7}$$

$$+E_{\alpha,\beta}(t^\alpha \lambda)p_{n-1}^\nu(t),\ n \geq 1, 0 \leq \nu \leq 1$$

According to Bailey (1954, 1990), $p_n^\nu(t)$ is the probability that there are n patients in the queue at time t and the probability generating function is $G^\nu(z,t)$, *i.e.*

$$G^\nu(z,t) = \sum_{n=0}^{\infty} z^{n\nu} p_n^\nu(t),\ |z| \leq 1 \tag{8}$$

and

$$\bar{G}^\nu(z,t) = \sum_{n=0}^{\infty} z^{n\nu} \bar{p}_n{}^\nu(t)$$

Multiplying equation (7) by $\sum_{n=0}^{\infty} z^{n\nu}$, $n = 0, 1, 2, \ldots,$ we get

$$\begin{aligned} \sum_{n=0}^{\infty} z^{n\nu} \frac{\delta^\nu p_n^\nu(t)}{\delta t^\nu} &= -E_{\alpha,\beta}(t^\alpha(\lambda+\mu)) \sum_{n=0}^{\infty} z^{n\nu} p_n^\nu(t) + E_{\alpha,\beta}(t^\alpha \mu) \sum_{n=0}^{\infty} z^{n\nu} p_{n+1}^\nu(t) \\ &\quad + E_{\alpha,\beta}(t^\alpha \lambda) \sum_{n=0}^{\infty} z^{n\nu} p_{n-1}^\nu(t) \\ \Longrightarrow \sum_{n=0}^{\infty} z^{n\nu} \frac{\delta^\nu p_n^\nu(t)}{\delta t^\nu} &= -E_{\alpha,\beta}(t^\alpha \lambda) \sum_{n=0}^{\infty} z^{n\nu} p_n^\nu(t) - E_{\alpha,\beta}(t^\alpha \mu) \sum_{n=0}^{\infty} z^{n\nu} p_n^\nu(t) \\ &\quad + E_{\alpha,\beta}(t^\alpha \mu) \sum_{n=0}^{\infty} z^{n\nu} p_{n+1}^\nu(t) + E_{\alpha,\beta}(t^\alpha \lambda) \sum_{n=0}^{\infty} z^{n\nu} p_{n-1}^\nu(t) \end{aligned}$$

And adding with equation (8), we get

$$\begin{aligned} \frac{d}{dt} G^\nu(z,t) &= \left(\frac{\mu}{z^{n\nu}} + E_{\alpha,\beta}(t^\alpha \lambda) z^{n\nu} - E_{\alpha,\beta}(t^\alpha(\lambda+\mu)) \right) (G^\nu(z,t) - p_0^\nu(t)) \\ &\quad + \frac{E_{\alpha,\beta}(t^\alpha \lambda)(z^{n\nu}-1)}{z^{n\nu}} p_0^\nu(t) \end{aligned} \tag{9}$$

Applying the Laplace transformation

$$\bar{G}^\nu(z,s) = \int_0^\infty e^{-st} p_n{}^\nu(z,t)\ dt$$

in equation (9), we get

$$\left(s^{\nu}\bar{G}^{\nu}(z,s)-s^{\nu-1}G^{\nu}(z,0)\right) = \left(\frac{E_{\alpha,\beta}(t^{\alpha}\mu)}{z^{n\nu}}+E_{\alpha,\beta}(t^{\alpha}\lambda)z^{n\nu}-E_{\alpha,\beta}(t^{\alpha}(\lambda+\mu))\right)\left(\bar{G}^{\nu}(z,s)-\bar{p_0}^{\nu}(s)\right)$$

where $\bar{p_0}^{\nu}(s)=\int_0^{\infty}e^{-st}p_0{}^{\nu}(t)\,dt$
After simplification, we get
$s^{\nu}\bar{G}^{\nu}(z,s)-s^{\nu-1}G^{\nu}(z,0)$

$$= \left(\frac{E_{\alpha,\beta}(t^{\alpha}\mu)}{z^{n\nu}}+E_{\alpha,\beta}(t^{\alpha}\lambda)z^{n\nu}-E_{\alpha,\beta}(t^{\alpha}\lambda)\right)\bar{G}^{\nu}(z,s)-\left(\frac{E_{\alpha,\beta}(t^{\alpha}\mu)}{z^{n\nu}}+E_{\alpha,\beta}(t^{\alpha}\lambda)z^{n\nu}-E_{\alpha,\beta}(t^{\alpha}\lambda)\right)\bar{p_0}^{\nu}(s)$$

$$\left\{s^{\nu}-\left(\frac{E_{\alpha,\beta}(t^{\alpha}\mu)}{z^{n\nu}}+E_{\alpha,\beta}(t^{\alpha}\lambda)z^{n\nu}-E_{\alpha,\beta}(t^{\alpha}\lambda)\right)\right\}\bar{G}^{\nu}(z,s) = s^{\nu-1}z^{n\nu i+1}-\left(\frac{E_{\alpha,\beta}(t^{\alpha}\mu)}{z^{n\nu}}+E_{\alpha,\beta}(t^{\alpha}\lambda)z^{n\nu}-E_{\alpha,\beta}(t^{\alpha}\lambda)\right)p_0{}^{\nu}$$

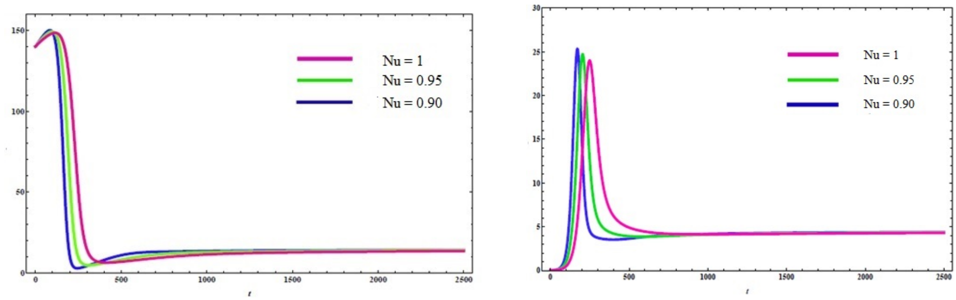
$$\Longrightarrow \bar{G}^{\nu}(z,s) = \frac{s^{\nu-1}z^{n\nu i+1}-\left(\frac{E_{\alpha,\beta}(t^{\alpha}\mu)}{z^{n\nu}}+E_{\alpha,\beta}(t^{\alpha}\lambda)z^{n\nu}-E_{\alpha,\beta}(t^{\alpha}\lambda)\right)p_0{}^{\nu}}{s^{\nu}-\left(\frac{E_{\alpha,\beta}(t^{\alpha}\mu)}{z^{n\nu}}+E_{\alpha,\beta}(t^{\alpha}\lambda)z^{n\nu}-E_{\alpha,\beta}(t^{\alpha}\lambda)\right)}$$

Now, $\bar{G}^{\nu}(z,s)$ converges in the region $|z|\leq 1$, the zero of the numerator and denominator of $\bar{G}^{\nu}(z,s)$ must coincide.

4 Numerical Example

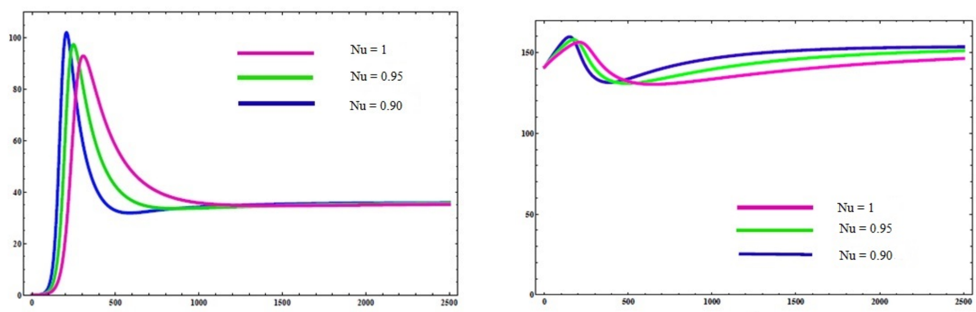
The results obtained below are implemented for estimating the number of arrivals of patients with severe diseases from different departments of public hospital under the assumption mentioned above.
For the numerical solutions of a system of fractional differential equations we use the real data sets, such as, a) the patients waiting time; b) the service time; c) number of patients with severe disease. The data has been collected directly from a public hospital by using the direct observational method. These data (collected for 500 patients) contains all the relevant information regarding each patient.
The simulated solution of the mean size of the arrival patients, the expected service rate, queue size and total patients in the system over the time are displayed in Figures (1)-(4) for $\nu=1,0.95,0.90$ and $\alpha=\beta=0.5$. ν represents server activity status. When $\nu=1$, it means the server is completely (100%) active. $\nu=0.95$ and 0.90 implies 95% and 90% of the server is active respectively. Further, we denote the

rate of arrival of patients who belong to non-severe and severe category as α and β.



(a) (b)

Figure 1: (a) The mean size of the arrival of patients during the time for different values of $\nu = 1, 0.95, 0.90$ and $\alpha = \beta = 0.5$ ($i.e.$ $E^{\nu}_{0.5,0.5}(t)$). (b) The expected service time for different values of ν.



(a) (b)

Figure 2: (a) The mean queue size of the arrival of patients during the time for different values of $\nu = 1, 0.95, 0.90$ and $\alpha = \beta = 0.5$ ($i.e.$ $E^{\nu}_{0.5,0.5}(t)$). (b) The total patients in the system over the time for different values of ν.

Figure (1) represents the mean size of the arrival of patients during time for different values of ν. Here, X-axis denotes the time in minutes and the number of arrivals of patients suffering from severe disease is represented by the Y-axis. Further, curves are derived by taking different values of $\nu = 1, 0.95, 0.90$ and $\alpha = \beta = 0.5$ ($i.e.$ $E^{\nu}_{0.5,0.5}(t)$) and it was observed that curves are upward increasing at a particular point of time and then start to decreases. After decreasing to a certain point of time, curves run parallel to time axis. Also, the graph reveals that the mean size of the arrival of the patient in the hospital from 0 to 120 minutes is high at $\nu = 0.90$ as compared to the value of $\nu = 0.95$ and $\nu = 0.90$.

Figure (2) depicts the relation between the expected service time and the number

of patients who are in queue for getting service under the different values of v. It can be observed that the service time corresponding to $v = 1$ is largest followed by that of $v = 0.95$ and $v = 0.90$, which also shows that service time increases to the other value of v and slowly decreases at a point of time.
Figure(3) shows the mean queue size of the patients suffering from severe disease during the time for different values of v. Here, it is observed that the queue size is stable for all values of v at a fixed point of time.
Figure (4) defines the total number of patients in the system over the time and shows that each curve peaks at a certain point of time and then starts downward slope. And if $v = 1$, the total number of patients remaining in the system is lesser than the value of $v = 0.95$ and $v = 0.90$.
Table(1) and Table(2) below shows the values of the properties of the queue when implemented to the real-life data.
As revealed from the tables below the pattern of mean arrival of patients, expected

Table 1: The mean arrival of patients and expected service times of $E^{v}_{0.5,0.5}(t)$ for different values of v at different time periods when $\alpha = \beta = 0.5$

Time	Mean arrival of patients			expected service time		
	v			v		
	1	0.95	0.90	1	0.95	0.90
0	28.517 ≈ 29	28.266 ≈ 28	28.015≈ 28	0.00	0.00	0.00
50	25.416≈ 25	25.290≈ 25	25.164≈ 25	2.24	4.42	3.49
150	23.021≈ 23	22.937≈ 23	22.85≈ 23	1.64	2.10	1.97
200	21.119≈ 21	21.057≈ 21	20.994≈ 21	1.36	1.58	1.53
250	19.566≈ 20	19.516≈ 20	19.46≈ 19	1.18	1.32	1.29
300	18.268≈ 18	18.226≈ 18	18.184≈ 18	1.06	1.16	1.14
350	17.163≈ 17	17.128≈ 17	17.092≈ 17	0.97	1.04	1.03
400	16.209≈ 16	16.178≈ 16	16.146≈ 16	0.90	0.96	0.95
450	15.375≈ 15	15.347≈ 15	15.319≈ 15	0.84	0.89	0.88
500	14.638≈ 15	14.613≈ 15	14.588≈ 15	0.80	0.85	0.86

service time, mean queue size and the total number of patients in the queue at time t conforms to the findings of the simulated data. Here, it is observed that the mean queue size of the developed queue model is not equal to zero as per the simulated results because in real-life patients arrive at the system substantially before the service starts. Henceforth, all values decrease with time and reach a cusp at $t = 450$.

Table 2: The mean queue size and total number of patients $E^{\nu}_{0.5,0.5}(t)$ for different values of ν at different time periods when $\alpha = \beta = 0.5$

Time	Mean queue size			Total number of patients		
	ν			ν		
	1	0.95	0.90	1	0.95	0.90
0	40.296≈ 40	40.045≈ 40	39.793≈ 40	39.642≈ 40	39.591≈ 40	39.551≈ 40
50	34.532≈ 35	34.406≈ 34	34.280≈ 34	34.205≈ 34	34.180≈ 34	34.159≈ 34
100	30.459≈ 30	30.375≈ 30	30.292≈ 30	30.241≈ 30	30.224≈ 30	30.211≈ 30
150	27.402≈ 27	27.339≈ 27	27.277≈ 27	27.239≈ 27	27.226≈ 27	27.216≈ 27
200	25.005≈ 25	24.955≈ 25	24.904≈ 25	24.874≈ 25	24.864≈ 25	24.856≈ 25
250	23.063≈ 23	23.021≈ 234	22.979≈ 23	22.954≈ 23	22.945≈ 23	22.939≈ 23
300	21.451≈ 21	21.415≈ 21	21.379≈ 21	21.357≈ 21	21.350≈ 21	21.344≈ 21
350	20.086≈ 20	20.055≈ 20	20.023≈ 20	20.004≈ 20	19.998≈ 20	19.993≈ 20
400	18.913≈ 19	18.885≈ 19	18.857≈ 19	18.841≈ 19	18.835≈ 19	18.831≈ 19
450	16.195≈ 16	16.174≈ 16	15.153≈ 15	16.140≈ 16	16.136≈ 16	16.133≈ 16
500	15.579≈ 16	15.560≈ 16	15.441≈ 15	15.729≈ 16	15.625≈ 16	15.522≈ 16

5 Conclusion

The subject of fractional calculus is as old as differential calculus, but remains unexplored outside its theoretical bounds. Here, we attempt to apply that concept to queueing theory and develop its properties on the simple Markovian model. The resultant characteristics of the proposed model are implemented both with simulated and real-life values. It is revealed that the concept put forward conforms to both and fits very well into the theory of queues, particularly when the server is not found to function as it should.

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