STATISTICS IN TRANSITION new series, March 2017
Vol. 18, No. 1, pp. 151–165

THE APPLICATION OF BŰHLMANN-STRAUB MODEL TO THE ESTIMATION OF NET PREMIUM RATES DEPENDING ON THE AGE OF THE INSURED IN THE MOTOR THIRD LIABILITY INSURANCE

Anna Szymańska[1]

ABSTRACT

One of the basic variables used in the process of tariff calculation of premiums in motor liability insurance is the age of the insured. In this type of insurance offered by insurers operating on the Polish market, this variable is taken into account in the ratemaking by discounts and increases in assigned premium, known as the net premiums rates. The aim of this work is to propose a method of rate estimation of net premiums in the groups of the motor third liability insurance portfolio of individuals created by the age of the insured. For the premium estimation, one of the maximum likelihood models, called the Bűhlmann-Straub model, was used.

Key words: a posteriori ratemaking, credibility theory, premium for a group of insurance contracts, motor third liability insurance

1. Introduction

In motor liability insurance the premium calculation process consists of two stages. The first, called a priori ratemaking, is the determination of net premiums, using actuarial methods (Ostasiewicz (ed.), 2000) based on certain risk factors, known as the basic ratemaking variables. The premium defined in this way, increased by the costs of insurance operations and the security addition among other things, is known as the base premium. The second stage of ratemaking is *a posterior* ratemaking, consisting in the base premium increases and discounts depending on individual risk factors of the insured. The bonus-malus system are one of the components of *the posterior* ratemaking commonly used in Europe. The bonus-malus systems (Lemaire, 1995, p. 3) differentiate the premium, with respect to the number of claims reported by the insured in the previous insurance period, which is based on the damage history of the insured. In addition to the

[1] Department of Statistical Methods, University of Lodz. E-mail:szymanska@uni.lodz.pl

bonus-malus system insurance companies may use other discounts and increases in the premium dependent on additional ratemaking variables, such as the age of the insured, the period of driving license holding, possession or not of children under the age of 12, profession of the insured, the age of the car, using car for business purposes, having or not any other insurance with the same company, continuation of insurance, etc. Competition on the insurance market should force the insurers to use different ratemaking variables in the ratemaking process to match premiums to the risk, represented by the insured, which should lead to lower premiums especially for those insured who do not cause damage. European countries use a few (usually from one to four) basic ratemaking variables. Most countries, including Poland, use the registration area of the vehicle and engine capacity as a major ratemaking factor pricing. In Europe, additional variables most often used in the ratemaking are the age of the insured, the use of the vehicle for commercial purposes and the age of the car. In Poland, an increase in the insured under the age of 25 is as high as 300% of the base premium. In some countries, such as France or Norway, the age is an element of *the prior* ratemaking. The aim of this paper is to present the method for determining the increases and discounts in the premium depending on the age of insured on the basis of estimation of premiums by the credibility estimation method and evaluation of premium rates related to the age applied in the audited insurance company. The author's proposition is to use the Bűhlmann-Straub model for this purpose. An example of the application of the new method is presented based on the data obtained from one of the insurance companies operating on the Polish market, which has reserved the right to stay anonymous.

2. Bűlmann-Straub model

Let X_{ij} denote the total amount of claims paid (or the number of claims) for the *i*-th insured (the *i*-th sub-group) in the *j*-th year of insurance. Suppose that the insurer has observations x_{ij}, $i=1,...,N$, $j=1,...,t$, which are the realizations of random variables X_{ij}. The amounts of payments $x_{i,t+1}$ in year $t+1$ are not known.

Let us assume that for each i the distribution of the random variable X_{ij} depends on parameter θ_i and that random variables X_{ij} by given $\Theta_i = \theta_i$ are independent and have the same distribution. Random vector $\mathbf{X}_i = (X_{i1},..., X_{it})$ denotes the individual history of insurance for the policy i (*i*-th sub-group) in a portfolio consisting of N policies (subgroups). The aim of the insurer is to determine the net premium in the year $t+1$ for the contract and the (*i*-th sub-group), given the vector $\mathbf{x}_i = (x_{i1},..., x_{it})$.

Assuming the equivalence of claims and premiums – net premium $m(\theta_i)$ for contract i (*i*-th sub-group) is defined by the formula:

$$m(\theta_i) = E\left(X_{i,t+1} \middle| \Theta_i = \theta_i\right) \tag{1}$$

Since we do not know θi parameter value, the value of net contributions is unknown.

The premium calculated as a weighted average from the premium for the entire portfolio, i.e. collective premiums $\mu = EX_{ij} = \frac{1}{Nt}\sum_{i=1}^{N}\sum_{j=1}^{t} x_{ij}$ and individual premium $\bar{x}_i = \frac{1}{t}\sum_{j=1}^{t} x_{ij}$ in the form:

$$m(\theta_i) = Z_i \bar{x}_i + (1 - Z_i)\mu \tag{2}$$

is called a credibility premium for the i-th contract (the i-th sub-group), where $Z_i \in [0,1]$ is a credibility factor (Kowalczyk, Poprawska and Ronka-Chmielowiec, 2006).

The estimator of variable $X_{i,t+1}$ is called a predictor of this variable, while the predictor's value is called a forecast for $X_{i,t+1}$ based on observations $x_{i1},..., x_{it}$. The basis of the credibility theory is the Bayesian statistical analysis with quadratic loss function (Krzyśko,1996).

One of the tasks of the credibility theory is to determine the values of the credibility Z_i factor. A small value of the coefficient means that the collective premium is more credible for the insurer than the individual premium. The factor Z_i is approximately equal to one when the history of damage to the policy or a group policy is long and has small variation with respect to time, or when contracts (group of policies) are very different from one another in terms of the history of damage.

Historically, the first model of the theory of credibility was the Bűhlmann model (Bűhlmann,1967), in which it is assumed that the portfolio policies can be divided into *N* sub-groups, each of which contains the same number of policies for which the data on *t* damage periods is available.

The Bűlmann-Straub model is a modified Bűlmann model, in which the number of policies included in the portfolio of individual subgroups does not have to be equal and which takes into account the importance of contracts in the portfolio. Also, the number of policies may vary periodically (Denuit, Marechal, Pitrebois and Walhin, 2007).

The model finds its application especially when a single policy or a small subset of policies differs significantly, in terms of risk profile, from the others. It is a one-way classification model. The model takes into account the weights (i.e. the volume of risk) w_{ij} of random variables X_{ij}. If the random variable X_{ij} denotes the arithmetic average of w_{ij} independent random variables with the same distributions, then the numbers w_{ij} are natural weights. The actuary, however, may establish its own weights, which do not have to be integers. In this model, insurance histories may have different lengths t_i for different contracts *i*. The structure of the data in the model is presented in Table 1 (Jasiulewicz, 2005).

Table 1. Structure of data in the Bűlmann-Straub model

Groups of policies	Years of insurance			
	1	2	...	t
1	x_{11} w_{11}	x_{12} w_{12}	...	x_{1t} w_{1t}
2	x_{21} w_{21}	x_{22} w_{22}	...	x_{2t} w_{2t}
...	...	...	...	...
N	x_{N1} w_{N1}	x_{N2} w_{N2}	...	x_{Nt} w_{Nt}

As previously assigned, let $\mathbf{X}_i = (X_{i1},..., X_{it})$ be a vector of observation of the number of damages for i-th policy (i-th subgroup of policies) during last t years, and let random variable Θ_i, represent the structure of risk in the portfolio.

The assumptions of the Bűlmann-Straub model (Bűlmann and Straub,1970):

1. For given i and $\Theta_i=\theta_i$, random variables $X_{i1},...,X_{it}$ are conditionally independent and

$$E(X_{ij}|\theta_i) = m(\theta_i), \tag{3}$$

$$Var(X_{ij} \; |\theta_i) = \frac{s^2(\theta_i)}{w_{ij}} \tag{4}$$

for i=1,...,N, j=1,...,t, wherein variables w_{ij} are known.

2. Pairs (Θ_1, $\boldsymbol{X}_1$),...,(Θ_N, $\boldsymbol{X}_N$) are mutually independent and random variables Θ_1,..., Θ_N are independent and have the same distributions.

Let there be given:

– the average amount of damages for the i-th sub-group of policies:

$$\overline{X}_{iw} = \sum_{j=1}^{t} \frac{w_{ij}}{w_i} X_{ij}, \quad w_i = \sum_{j=1}^{t} w_{ij} , \tag{5}$$

– the average amount of damage for the portfolio:

$$\overline{X}_{ww} = \sum_{i=1}^{N} \frac{w_i}{w} \overline{X}_{iw}, \quad w = \sum_{i=1}^{N} w_i , \tag{6}$$

– structural parameters of risk in the portfolio:

$$\mu = Em(\Theta_i) = EX_{ij}, \quad \varphi = Es^2(\Theta_i), \quad \psi = Var(m(\Theta_i)), \tag{7}$$

where:

- μ – collective net premium, which is a weighted average of the individual net premiums $m(\theta_i)$; the overall mean, it is the expected value of the claim amount for an arbitrary policyholder in the portfolio
- φ – describes the average volatility of claims in a group (variation within the group)
- ψ – describes the variation of claims between groups.

It can be proved that if the assumptions of the Bűlmann-Straub model are met, then (Kass, Goovaerts, Dhaene and Denuit, 2001):

1. best inhomogeneous linear predictor $\tilde{m}_i = E(X_{i\,n+1} | \mathbf{X}_i)$ of the credibility premium $m(\Theta_i)$ in the sense of least mean square error is of the form:

$$\tilde{m}_i = Z_i \overline{X}_{iw} + (1 - Z_i)\mu, \tag{8}$$

where the trust factor is $Z_i = \dfrac{w_i \psi}{w_i \psi + \varphi}$

2. best homogeneous linear predictor $\tilde{m}_i^*$ of the credibility premium $m(\Theta_i)$ in the sense of least mean square error is of the form:

$$\tilde{m}_i^* = Z_i \overline{X}_{iw} + (1 - Z_i)\overline{X}_{zw}, \tag{9}$$

where the trust factor is $Z_i = \dfrac{w_i \psi}{w_i \psi + \varphi}$ and $\overline{X}_{zw} = \sum_{i=1}^{N} \dfrac{Z_i}{Z} \overline{X}_{iw}, \quad Z = Z_1 + \ldots + Z_N$.

It can be proved that if the assumptions of the Bűlmann-Straub model are met, then unbiased estimators of structural parameters in the portfolio are of the form (Kass, Goovaerts, Dhaene, and Denuit, 2001):

$$\hat{\mu} = \overline{X}_{zw}, \ \hat{\varphi}_N = MSW, \ \hat{\psi} = \frac{w(N-1)(MSB - MSW)}{w^2 - \sum_{i=1}^{N} w_i^2}, \tag{10}$$

where:

$SSW = \sum_{i=1}^{N} \sum_{j=1}^{t} w_{ij} (X_{ij} - \overline{X}_i)^2$ - the weighted sum of squares of deviations within groups (*sum-of-squares-within*);

$MSW = \frac{SSW}{(t-1)N}$ - the average weighted sum of squares of deviations within groups (*mean-square-within*);

$SSB = \sum_{i=1}^{N} w_i (\overline{X}_{iw} - \overline{X}_{ww})^2$ - the weighted sum of squares of deviations between groups (*sum-of-squares-between*);

$MSB = \frac{SSB}{N-1}$ - the average weighted sum of squares of deviations between groups (*mean-square-between*).

If the assumptions of the Bűlmann-Straub model are met, the average square error of inhomogenous and homogeneous predictor of credibility premium $m(\Theta_i)$ are respectively (Daykin, Pentikäinen and Pesonen,1994):

$$MSE_i = E(m(\Theta_i) - \tilde{m}_i)^2 = (1 - Z_i)\psi , \qquad (11)$$

$$MSE_i^* = E(m(\Theta_i) - \tilde{m}_i^*)^2 = (1 - Z_i)\psi \left(1 + \frac{1 - Z_i}{Z} \right) \qquad (12)$$

for $i = 1,..., N$.

3. Example of application of the model to evaluate the rates of premium in groups separated by the age of insured

An empirical research was carried out based on the data from the portfolio of the third party liability insurance of motor vehicle owners individuals from the period of four years. For the sake of the study more than 100,000 policies were drawn for each year analyzed (the exact sample size is not specified due to the anonymity of the data). In what follows, this sample will be called portfolio. The data, in an aggregated form, on the number and value of claims paid with respect to the age groups of the insured are presented in Tables 2 and 3. The division of the insured into age groups is consistent with the classification of the insurer. The specified number of claims and the division into classes according to the value of claims paid is consistent with the tariffs of the insurer.

Table 2. The structure of the insured by the age and the number of claims paid in the motor third liability insurance portfolio in the years analyzed [%]

year	age	number of claims				portfolio
		0	1	2	3	
1	18-25	1.2560	0.0889	0.0049	0.0005	1.3503
	25-28	2.5665	0.1186	0.0049	0.0000	2.6899
	28-43	41.6109	1.6970	0.0674	0.0038	43.3790
	43-53	27.0127	1.1649	0.0361	0.0032	28.2170
	53-90	23.3994	0.9299	0.0323	0.0022	24.3638
	∑	95.8455	3.9993	0.1455	0.0097	100.0000
2	18-25	1.0846	0.0869	0.0040	0.0004	1.1758
	25-28	2.4324	0.1145	0.0047	0.0007	2.5524
	28-43	42.3952	1.7107	0.0625	0.0040	44.1724
	43-53	25.5445	1.0824	0.0385	0.0018	26.6672
	53-90	24.4352	0.9617	0.0345	0.0007	25.4321
	∑	95.8918	3.9562	0.1443	0.0076	100.0000
3	18-25	1.3342	0.1013	0.0052	0.0005	1.4411
	25-28	2.8953	0.1418	0.0079	0.0007	3.0457
	28-43	42.5567	1.7898	0.0611	0.0025	44.4100
	43-53	23.8434	1.0471	0.0486	0.0017	24.9408
	53-90	25.0821	1.0385	0.0397	0.0020	26.1624
	∑	95.7117	4.1185	0.1624	0.0074	100.0000
4	18-25	0.9935	0.0675	0.0063	0.0000	1.0672
	25-28	2.0259	0.0983	0.0048	0.0003	2.1292
	28-43	42.5934	1.5122	0.0549	0.0020	44.1626
	43-53	23.3443	0.8661	0.0344	0.0005	24.2452
	53-90	27.4171	0.9433	0.0336	0.0018	28.3958
	∑	96.3742	3.4874	0.1339	0.0045	100.0000

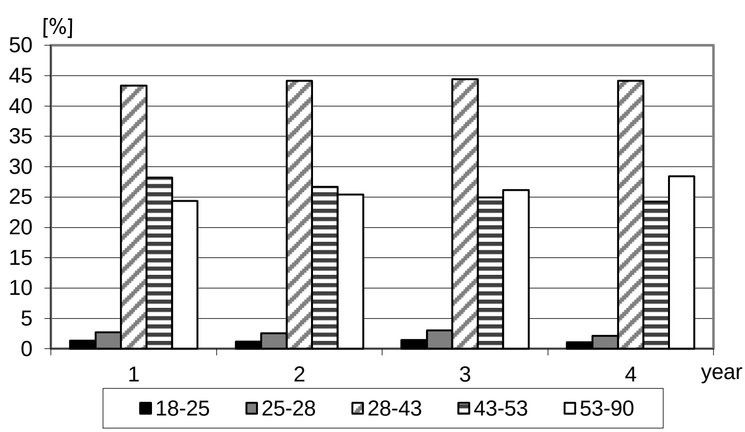

Figure 1. The structure of the insured by the age in the motor third liability insurance portfolio in the years analyzed [%]

In the years analyzed, people aged under 25 accounted for just over 1% of the insured in the analyzed portfolio. The largest group are the insured at the age from 28 to 43 (approx. 44%). Figure 2 presents the mean number of claims paid in each age group in the years analyzed. The highest loss ratio of above 0.07 in the studied period was observed in the group of under 25. Among the insured aged from 25 to 28 there has been an annual average of 0.048 to 0.052 damages. It should be noted that in other age groups, the average number of claims submitted per year was smaller, it varied from 0.036 to 0.046 and was close to the portfolio mean, which ranged from 0.038 to 0.45.

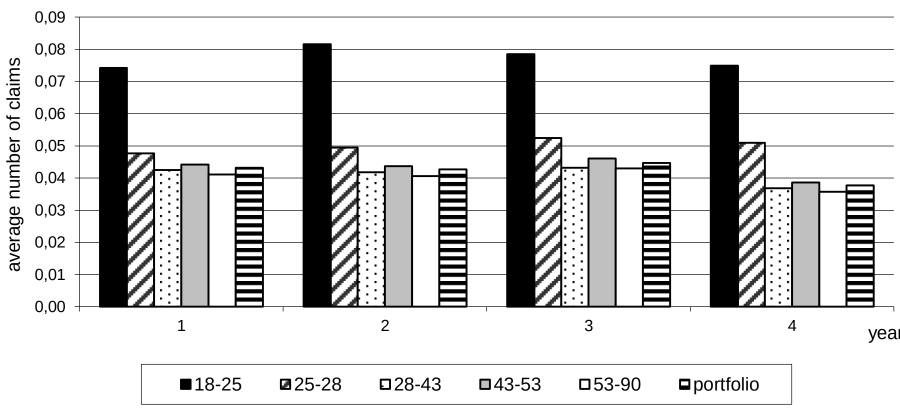

Figure 2. The average number of claims paid in the motor third liability insurance portfolio in the years analyzed according to the insured age groups

Table 3. The structure of the insured by the age and the value of claims paid in the motor third liability insurance portfolio in the years analyzed

year	age [years]	value of claim [thousands of zlotys]											portfolio
		(0,1]	(1,3]	(3,5]	(5,10]	(10,20]	(20,30]	(30,40]	(40,50]	(50,100]	(100,200]	> 200	
1	18- 25	0.31	0.65	0.39	0.52	0.23	0.09	0.03	0.00	0.04	0.01	0.00	2.27
	25-28	0.58	1.25	0.45	0.34	0.16	2.81	0.01	0.01	0.08	0.00	0.00	2.98
	28-43	8.99	17.10	6.63	5.64	2.21	16.97	0.44	0.19	0.48	0.12	0.01	42.55
	43-53	5.73	12.07	4.69	3.50	1.66	0.84	0.26	0.09	0.40	0.05	0.01	29.01
	53-90	4.42	9.68	3.65	3.11	1.33	1.13	0.14	0.06	0.30	0.04	0.01	23.20
	∑	20.04	40.75	15.82	13.10	5.59	5.33	0.88	0.36	1.30	0.15	0.04	100.00
2	up to 25	0.27	0.78	0.33	0.40	0.26	0.08	0.02	0.03	0.03	0.02	0.00	2.21
	25-28	0.49	1.04	0.48	0.47	0.25	0.12	0.04	0.02	0.03	0.00	0.00	2.92
	28-43	7.74	18.29	6.67	5.78	2.60	1.05	0.35	0.19	0.44	0.12	0.04	43.27
	43-53	5.49	10.95	4.47	3.32	1.73	0.74	0.20	0.13	0.22	0.06	0.02	27.34
	53-90	4.89	9.73	3.98	3.06	1.51	0.50	0.21	0.14	0.22	0.02	0.01	24.26
	∑	18.88	40.78	15.93	13.02	6.35	2.49	0.82	0.50	0.94	0.21	0.07	100.00

Table 3. The structure of the insured by the age and the value of claims paid in the motor third liability insurance portfolio in the years analyzed (cont.)

year	age [years]	value of claim [thousands of zlotys]											portfolio
		(0,1]	(1,3]	(3,5]	(5,10]	(10,20]	(20,30]	(30,40]	(40,50]	(50,100]	(100,200]	> 200	
3	up to 25	0.40	0.84	0.45	0.42	0.23	0.06	0.05	0.02	0.02	0.01	0.01	2.50
	25-28	0.49	1.48	0.59	0.48	0.25	0.07	0.04	0.05	0.05	0.01	0.01	3.51
	28-43	7.34	17.90	7.16	5.89	2.82	1.05	0.42	0.19	0.31	0.11	0.02	43.23
	43-53	4.29	10.97	4.00	3.38	1.74	0.55	0.23	0.12	0.23	0.05	0.02	25.60
	53-90	4.59	10.46	4.22	3.39	1.39	0.53	0.26	0.09	0.19	0.03	0.01	25.17
	∑	17.11	41.66	16.42	13.55	6.43	2.27	1.01	0.48	0.81	0.21	0.06	100.00
4	do 25	0.30	0.75	0.34	0.37	0.17	0.05	0.02	0.01	0.01	0.01	0.01	2.03
	25-28	0.42	1.00	0.64	0.44	0.18	0.10	0.03	0.01	0.02	0.00	0.00	2.84
	28-43	8.53	17.74	7.56	5.08	2.90	0.66	0.37	0.12	0.26	0.03	0.02	43.28
	43-53	4.79	10.13	4.30	3.13	1.73	0.42	0.11	0.03	0.14	0.03	0.03	24.84
	53-90	5.61	10.91	4.71	3.27	1.67	0.38	0.12	0.10	0.19	0.02	0.02	27.00
	∑	19.66	40.54	17.56	12.28	6.64	1.61	0.65	0.27	0.62	0.10	0.08	100.00

Figure 3 presents the average value of claims paid in separate age groups of the insured.

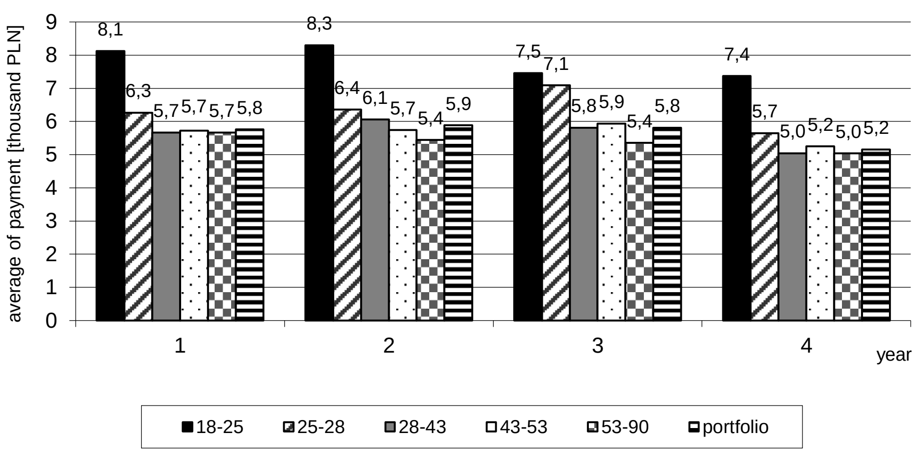

Figure 3. The average value of claims paid in the motor third liability insurance portfolio in the years analyzed, according to the age of the insured

As one can see in Figure 3, the highest average value of payments was observed in the group of insured under the age of 25. Analyzing the data from Table 3, one can observe that the structure of claims paid in the group of insured persons under the age of 25 differs from the structure of payments in other age groups. Persons under the age of 25 cause less damage of low value and more damage of higher value. For example, the payout structure of different age groups surveyed for the first years is shown in Figure 4. Damages up to 5 thousand zlotys constitute 59% of all payments in the group of insured persons under the age of 25, in other age groups such damages constitute approx. 77% of the claims paid. At the same time the group of insured persons under the age of 25 has a greater share of the compensation values from 5 thousand zlotys to 10 thousand zlotys (23% of payments in this group) and from 10 thousand zlotys to 20 thousand zlotys (10% of payments in this group), in other age groups it is approx. 12% and 6% of payments, respectively.

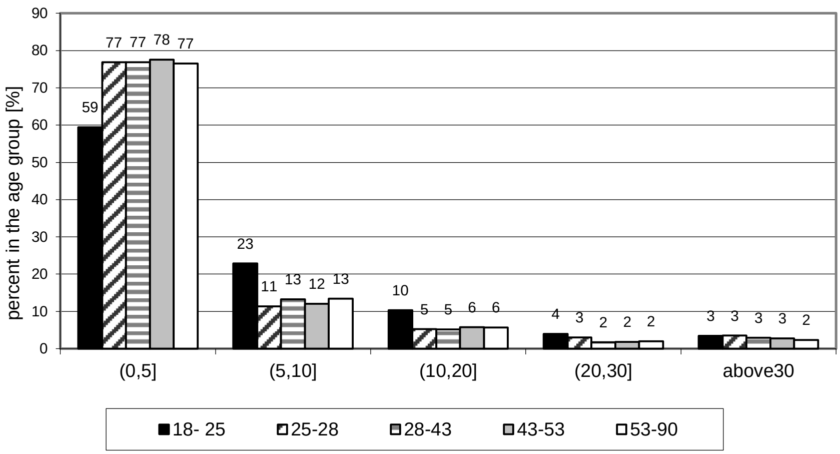

Figure 4. The structure of claims paid in the motor third liability insurance portfolio in the years analyzed, according to the age of the insured

The analysis of the number and the value of claims paid confirms the justification of taking into account the age of the insured in the ratemaking, especially for drivers aged up to 25. In motor insurance, the individual net premium in the period t+1 is determined by the equation (Szymańska, 2014):

$$\Pi(X,K) = (EX)\cdot(EK)\cdot b_{t+1} \qquad (13)$$

where $\Pi(X,K)$- individual net premium in period $t+1$, EX – the expected value of a single loss in the portfolio, EK – the expected number of claims for individual insurance portfolio, b_{t+1} – the rate of premium in period $t+1$. In actuarial literature the independence of random variables of the amount and number of damages is assumed.

The aim of this study is to determine the coefficient b_{t+1} constituting the increase or discount of the premium dependent on the age of the insured. Net premiums were estimated using the Bűlmann-Straub model. Models based on the theory of credibility do not require assumptions about the form of the random variable describing the size of individual loss in the portfolio and the values of the parameters of this distribution.

The credibility premium is determined by multiplying the expected value of the estimated payments by the Bűlmann-Straub model in particular age groups of the insured (based on data from Table 4), and the expected number of claims is also estimated based on the Bűlmann-Straub model (see the data in Table 6). Two cases were considered: when the contribution of the credibility is a heterogeneous or homogeneous predictor of the net premium. Tables 5 and 7 show the results of the estimation. Premium rates (${}_ib_{t+1}$ and ${}_ib^*_{t+1}$) in different age groups were calculated as the ratio of the credibility premiums in a given age group and the credibility premium in the portfolio:

$$ {}_ib_{t+1} = \frac{\tilde{m}_i \cdot {}^K\tilde{m}_i}{\tilde{m}_{portf} \cdot {}^K\tilde{m}_{portf}} \cdot 100\% \tag{14} $$

$$ {}_ib^*_{t+1} = \frac{\tilde{m}^*_i \cdot {}^K\tilde{m}^*_i}{\tilde{m}^*_{portf} \cdot {}^K\tilde{m}^*_{portf}} \cdot 100\% \tag{15} $$

The value of the credibility premiums, premium rates and net premiums is presented in Table 8.

Table 4. The average value of compensation paid [thousand zlotys] in the portfolio according to the age of the insured in the years analyzed

i (age group)	*j* (year)							
	1		2		3		4	
	X_{ij}	w_{ij} [%]	X_{ij}	w_{ij} [%]	X_{ij}	w_{ij} [%]	X_{ij}	w_{ij} [%]
1	8.12571	2.27	8.30120	2.21	7.45528	2.50	7.37585	2.03
2	6.26419	2.98	6.36364	2.92	7.08891	3.51	5.65085	2.84
3	5.66982	42.55	6.06428	43.27	5.81042	43.23	5.04389	43.28
4	5.72357	29.01	5.74814	27.34	5.93312	25.60	5.24798	24.84
5	5.66751	23.20	5.44195	24.26	5.35467	25.17	5.04204	27.00

X_{ij} – the average value of claim paid in *i*-th group in period *j* [thousand zlotys],

w_{ij} – the share of policies in *i*-th group of the portfolio in period *j* [%],

1 – group of the insured at the age of 18-25, 2 – group of insured at the age of 25-28, 3 – group of insured at the age of 28-43, 4 – group of insured at the age of 43-53, 5 – group of insured at the age of 53-90.

Table 5. Coefficients of credibility, credibility premium [thousand zlotys] and estimation errors according to age groups

i	Z_i	$\tilde{m}_i^*$	$\tilde{m}_i$	MSE_i^*	MSE_i
1	0.34	6.51136	6.35357	0.13214	0.11071
2	0.42	6.11444	5.97458	0.11497	0.09814
3	0.91	5.65019	5.62833	0.01575	0.01534
4	0.86	5.70044	5.66652	0.02479	0.02380
5	0.85	5.40268	5.36751	0.02575	0.02468

Table 6. The average number of claims in the portfolio according to the age of the insured in the years analyzed

i (age group)	j (year)							
	1		2		3		4	
	K_{ij}	w_{ij}	K_{ij}	w_{ij}	K_{ij}	w_{ij}	K_{ij}	w_{ij}
1	0.0743	1.3503	0.0816	1.1758	0.0785	1.4411	0.0750	1.0672
2	0.0477	2.6899	0.0494	2.5524	0.0524	3.0457	0.0510	2.1292
3	0.0425	43.3790	0.0418	44.1724	0.0432	44.4100	0.0369	44.1626
4	0.0442	28.2170	0.0437	26.6672	0.0461	24.9408	0.0386	24.2452
5	0.0411	24.3638	0.0406	25.4321	0.0430	26.1624	0.0358	28.3958

K_{ij} – the average number of claims in the i-th group in period j ,

w_{ij} – the share of policies in i-th group of the portfolio in period j [%],

1 – group of insured at the age of 18-25, 2 – group of insured at the age of 25-28, 3 – group of insured at the age of 28-43, 4 – group of insured at the age of 43-53, 5 – group of insured at the age of 53-90.

Table 7. Coefficients of credibility, the number of damages estimated by means of the credibility method and estimation errors for age groups of the insured

i	Z_i	${}^{K}\tilde{m}_i^*$	${}^{K}\tilde{m}_i$	MSE_i^*	MSE_i
1	0.38	0.0582	0.0554	0.0000192	0.0000164
2	0.56	0.0488	0.0468	0.0000131	0.0000117
3	0.96	0.0411	0.0409	0.0000012	0.0000012
4	0.93	0.0433	0.0429	0.0000020	0.0000020
5	0.93	0.0403	0.0399	0.0000019	0.0000019

Taking into account equations (13), (8) and (9), the value of the net premium was determined from the formulas:

$$\Pi_i(X,K) = \tilde{m}_i \cdot {}^{K}\tilde{m}_i \cdot {}_{i}b_{t+1} \tag{16}$$

$$\Pi_i^*(X,K) = \tilde{m}_i^* \cdot {}^{K}\tilde{m}_i^* \cdot {}_{i}b_{t+1}^* \tag{17}$$

The value of the credibility premiums, premium rates and net premiums is presented in Table 8.

Table 8. Net premiums and contributions of net premiums according to the age of the insured.

Age [years]	Credibility premium [thousand zlotys]		Premium rates		Net premium [PLN]	
	$\tilde{m}_i^* \cdot {}^{K}\tilde{m}_i^*$	$\tilde{m}_i \cdot {}^{K}\tilde{m}_i$	${}_{i}b_{t+1}^*$	${}_{i}b_{t+1}$	$\Pi_i^*(X,K)$	$\Pi_i(X,K)$
18-25	0.37867	0.35186	1.3920	1.4931	527.13	525.37
25-28	0.29818	0.27958	1.0961	1.1864	326.84	331.69
28-43	0.23196	0.22995	0.8527	0.9758	197.79	224.38
43-53	0.24655	0.24321	0.9063	1.0321	223.46	251.00
53-90	0.21749	0.21435	0.7995	0.9096	173.88	194.97
portfolio	0.27203	0.23565	1.0000	1.0000	272.03	235.65

4. Conclusions

The analysis of the number and the value of claims paid in the analyzed portfolio justifies the use of the age of the insured as one of the ratemaking variables. Insured persons aged from 18 to 25 cause on average more damage per year with higher average value, and therefore should pay higher premiums. The estimation results indicate a contribution rate of between 140% and 150% of the basic premium. In the analyzed insurance company, in the years investigated, the insured under the age of 25 pay 300% of the basic premium if they buy insurance for the first time in this company, and 200% of the basic premium on the continuation of insurance. Also, people aged 25 to 28, cause on average more damage with the value slightly exceeding the average value. According to the estimation method, they should pay a contribution rate of 119%. In the insurance company analyzed, for drivers aged 25 to 28 who took out insurance for the first time in the company paying the rate of 170% of the basic premium, the rate on the continuation of insurance was 130% of the premium. Another group that should have raised contributions are the insured at the age from 43 to 53, according to estimates. Insured persons in this age group should pay premiums increased by 4%. In the studied company they were at a 10% rise in the premium unless they signed a declaration about not sharing a vehicle with any person aged up to 25. The study reveals that persons aged 28-43 and over 53 could have a small discount. However, in the analyzed insurance company there were no discounts due to the age of the insured. There was also no discount due to the time of the possession of a driving license.

REFERENCES

BÜHLMANN, H., (1967). Experience Rating and Credibility, ASTIN Bulletin, 4 (3), pp. 199–207.

BÜHLMANN, H., STRAUB, E., (1970). Glaubwürdigkeit für Schadensätze, Mitteilungen der Vereiningung scheizerischer Vesicherungsmathematiker, 1970, pp. 111–133.

DAYKIN, C. D., PENTIÄINEN, T., PESONEN, M., (1994). Practical Risk Theory for Actuaries, Chapman & Hall, London.

DENUIT, M., MARECHAL, X., PITREBOIS, S., WALHIN, J. F., (2007). Actuarial Modelling of Claim Counts. Risk classification, Credibility and Bonus-Malus Systems, J. Wiley & Sons, England.

JASIULEWICZ, H., (2005). Teoria zaufania. Modele aktuarialne, Wydawnictwo AE im. Oskara Langego we Wrocławiu, Wrocław.

KAAS, R., GOOVAERTS, M., DHAENE, J., DENUIT, M., (2001). Modern Actuarial Risk Theory, Kluwer, Boston.

KRZYŚKO, M., (1996). Statystyka Matematyczna, Wydawnictwo Naukowe UAM, Poznań.

KOWALCZYK, P., POPRAWSKA, E., RONKA-CHMIELOWIEC, W., (2006). Metody aktuarialne, PWN, Warszawa.

LEMAIRE, J., (1995). Bonus-Malus Systems in Automobile Insurance, Kluwer, Boston.

OSTASIEWICZ, W., (red.), (2000). Modele Aktuarialne, Wydawnictwo Akademii Ekonomicznej im. O. Langego we Wrocławiu, Wrocław.

SZYMAŃSKA, A., (2014). Statystyczna analiza systemów bonus-malus w ubezpieczeniach komunikacyjnych, Wydawnictwo UŁ, Łódź.