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TRIANGULAR METHOD OF SPATIAL SAMPLING

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ABSTRACT

In this paper a new adaptive method of spatial sampling - a triangular method of spatial sampling is presented. The theory of this method is developed. Benefits of decreased size of a sample, when this method is used, are discussed. Initial sampling of the first three elements is described and density of sampling at the initial stage is obtained by Monte Carlo method. The density is defined on the basis of the logarithm of inverse square of the Euclidean distance function. Simulation of the triangular method of spatial sampling is conducted. An example is research on a forest. The aim of this research is to approximate the ability of trees to absorb carbon dioxide. In this example the triangular method of spatial sampling is used at the strata sampling stage. Density of sampling in the simulated forest is obtained using Monte Carlo method.

Key words: adaptive sampling, spatial sampling, stratified sampling, research on a forest, minimizing costs of sampling.

1. Sampling based on triangles

In adaptive sampling probabilities of inclusion depend on values of the variable of interest. Adaptive sampling is primarily used in the studies of rare phenomena and agglomerations of the characteristics under study on a relatively large area. This method allows, in such a case, to increase the probability of inclusion of elements which are near to the element with given characteristics. This feature gives the researcher some control over sample composition.

Several authors have dealt with the problem of selection of the elements with specified characteristics. For instance, Thompson and Seber [1996] describe searching for the people infected by a rare disease. If the infected person is found using simple random sampling, then automatically all people

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close to the infected person are also selected to the sample. Similar methods are applied in various fields of science. In this paper a new method of spatial sampling is constructed. Theoretical construction of the sampling method based on triangles is put in the context of the simulation of the ability of forest to absorb the carbon dioxide.

2. The method of sampling the first triangle

At the first stage three elements are selected. Let us denote $X1$ and $X2$ by some characteristics of population under study. Let us consider the realization of spatial sampling from space $X1 \times X2$. Three circular areas with a fixed radius are used as a base of the estimation. However, the selection will be conducted by sampling points which are the centres of circles. The first element is taken uniformly from the space $X1 \times X2$. In this way, none of the fragments of the forest is preferred. Let us denote the first sampled element by $(x1_1, x2_1)$. This element becomes a central point for sampling the next two elements. In order to draw next two elements, let us define distance function from point $(x1_1, x2_1)$ on the space $X1 \times X2$:

$$g(x1, x2) = dist((x1, x2), (x1_1, x2_1)), \tag{1}$$

where $dist$ is a function which satisfies the definition of distance function (satisfies the conditions of non-negativity, identity of indiscernibles, symmetry and subadditivity) and is integrable. When the $dist$ is selected, the specificity of the space $X1 \times X2$ should be taken into account. The g function is used to define density function on the space $X1 \times X2$, which is as follows:

$$f(x1, x2) = \begin{cases} (\alpha g(x1, x2))^{-1}, \text{ when } (x1, x2) \neq (x1_1, x2_1), \\ 0, \text{ when } (x1, x2) = (x1_1, x2_1), \end{cases} \tag{2}$$

where $\alpha = \iint_{X1 \times X2} g(x1, x2)^{-1} dx1dx2$. Sampling of X_1-coordinate is independent from sampling X_2-coordinate, thus the g function can be defined separately for each subspace:

$$\begin{aligned} g_1(x1) &= \text{dist}(x1, x1_1), \\ g_2(x2) &= \text{dist}(x2, x2_1). \end{aligned} \tag{3}$$

The density function can be also defined separately for each subspace:

$$f_1(x1) = \begin{cases} (\alpha_1 g_1(x1))^{-1}, \text{ when } x1 \neq x1_1, \\ 0, \text{ when } x1 = x1_1, \end{cases}$$
$$f_2(x2) = \begin{cases} (\alpha_2 g_2(x2))^{-1}, \text{ when } x2 \neq x2_1, \\ 0, \text{ when } x2 = x2_1, \end{cases} \tag{4}$$

where $\alpha_1 = \int_{X1} g_1(x1)^{-1} dx1$ and $\alpha_2 = \int_{X2} g_2(x2)^{-1} dx2$.

Finally, we define (using independence of sampling of each coordinate) the density function $f(x1, x2)$ on space $X1 \times X2$ as a product $f_1(x1) \cdot f_2(x2)$.

Next, two elements are sampled with probabilities defined by density function $f(x1, x2)$. The sampling plan of sample

$$s = \{K(x1_1, x2_1), K(x1_2, x2_2), K(x1_3, x2_3)\}, \tag{5}$$

where $K(x1_i, x2_i)$ denotes a circle centered at the point $(x1_i, x2_i)$, is as follows:

$$\begin{aligned} P(s) = c \Bigg[&\iint_{K(x1_2,x2_2)} f_1(x1,x2)dx1dx2 \iint_{K(x1_3,x2_3)} f_1(x1,x2)dx1dx2 \\ &+ \iint_{K(x1_1,x2_1)} f_2(x1,x2)dx1dx2 \iint_{K(x1_3,x2_3)} f_2(x1,x2)dx1dx2 \\ &+ \iint_{K(x1_1,x2_1)} f_3(x1,x2)dx1dx2 \iint_{K(x1_2,x2_2)} f_3(x1,x2)dx1dx2 \Bigg], \end{aligned} \tag{6}$$

where c is a ratio of the area of the circle to the area of entire space $X1 \times X2$, and f_i, $i = 1, 2$, denotes density functions defined by the equations (4) in the case when the first sampled element is a circle centred at the point $(x1_i, x2_i)$, $i = 1, 2, 3$.

Let us consider, for example, sampling of three elements in the way described above. Let $X1 \times X2$ be a plane $[0, 1] \times [0, 1]$. After determining on the plane $X1 \times X2$ the uniform distribution, the element with coordinates $(x1_1, x2_1)$ is sampled. The g function is defined as a product of g_1 and g_2 functions. These functions are as follows:

$$g_1(x1) = \begin{cases} \log((x1 - x1_1)^{-2}), \text{ when } x1 \neq x1_1, \\ 0, \text{ when } x1 = x1_1, \end{cases}$$
$$g_2(x2) = \begin{cases} \log((x2 - x2_1)^{-2}), \text{ when } x2 \neq x2_1, \\ 0, \text{ when } x2 = x2_1. \end{cases} \tag{7}$$

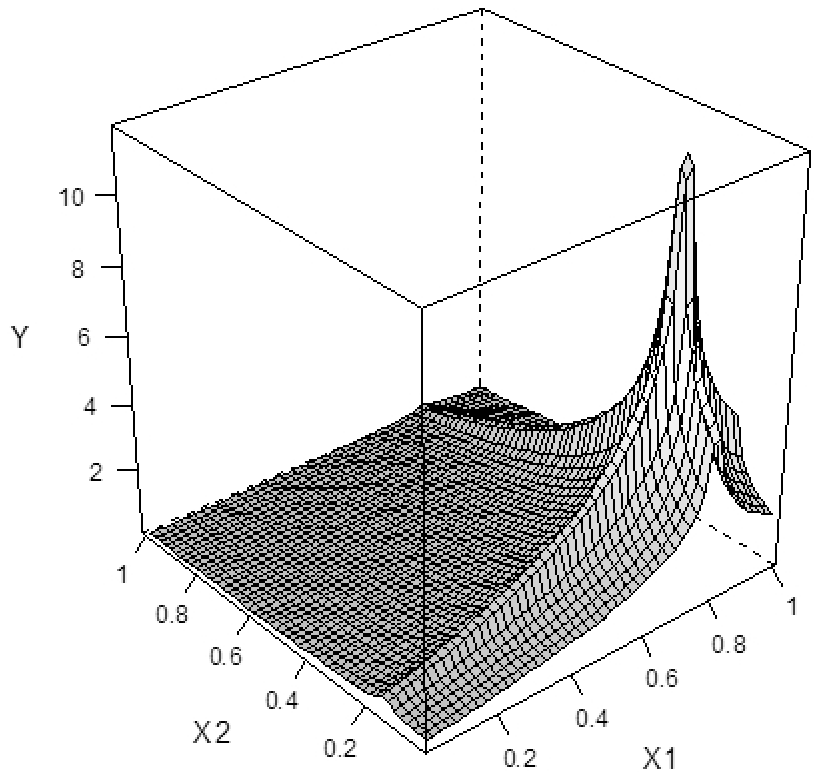

Figure 1. Density function defined on the basis of the logarithm of inverse square of the distance from the point (0.8026372,0.1422940).

Then, the probability of selecting the next element is given by the density function on $X_1 \times X_2$, which is a product of densities on X_1 and X_2. Density functions on spaces $X1$ and $X2$ are as follows:

$$f_1(x1) = \begin{cases} (\alpha_1 \log((x1 - x1_1)^{-2}))^{-1}, \text{ when } x1 \neq x1_1, \\ 0, \text{ when } x1 = x1_1, \end{cases}$$
$$f_2(x2) = \begin{cases} (\alpha_2 \log((x2 - x2_1)^{-2}))^{-1}, \text{ when } x2 \neq x2_1, \\ 0, \text{ when } x2 = x2_1, \end{cases} \tag{8}$$

where
$\alpha_1 = \int_{X1} \log((x1 - x1_1)^{-2})^{-1} dx1$ and $\alpha_2 = \int_{X2} \log((x2 - x2_1)^{-2})^{-1} dx2$
Figure 1 presents a density function in a situation when the first sampled element has coordinates $(x1_1, x2_1) = (0.8026372, 0.1422940)$.

A more fundamental problem than determing the probability of sampling the second and the third element, when the first is already sampled, is to determine the probability of sampling without any assumption about the elements already sampled. Such analysis was performed for the example presented above. Let us denote density function which determines this probability by $f_0(x1, x2)$. In order to approximate this function, sampling of the first element, defining the density function $f(x1, x2)$ and sampling of the second and the third elements were repeated 10 000 times. The density function $f_0(x1, x2)$ was defined as:

$$f_0(x1, x2) = \frac{1}{10000} \sum_{i=1}^{10000} f_i(x1, x2), \quad (9)$$

where functions $f_i(x1, x2)$, $i = 1, \dots, 10000$ are sequentially created densities defined as a product of the function (8). Approximation of density function $f_0(x1, x2)$ was obtained therefore by Monte Carlo method. The density function $f_0(x1, x2)$ could be used to determine the inclusion probabilities for initial sampling (sampling of the first triangle). The inclusion probabilities for initial sampling are

$$\pi_{K(x1_0,x2_0)} = \iint_{K(x1_0,x2_0)} f_0(x1, x2)dx1dx2,\ K(x1_0, x2_0) \in X_1 \times X_2. \quad (10)$$

Empirical inclusion probabilities defined in such way may be adopted in the Horvitz-Thompson statistics as well as in the variance estimators instead of their true counterparts [4]. Function $f_0(x1, x2)$ is presented in figure 2.

Figure 2 confirms that at the initial sampling stage (sampling of vertices of the first triangle) it is more likely to sample elements which are located more centrally. Of course, the shape of the density function $f_0(x1, x2)$ can be changed by another choice of distance function.

3. The method of sampling the subsequent triangles

Let us denote the characteristic under study by Z and the characteristic strongly correlated with the characteristic under study by Y. Let $\bar{y}$ be an average value of Y characteristic. The sampling method described below allows (of course with a certain lack of precision) the values of the elements which are included in the sample in next steps to be controlled. This sampling method is a kind of adaptive sampling. As in other adaptive sampling procedures, initial elements need to be chosen. The procedure of the triangular sampling method requires three initial elements. They form the vertices

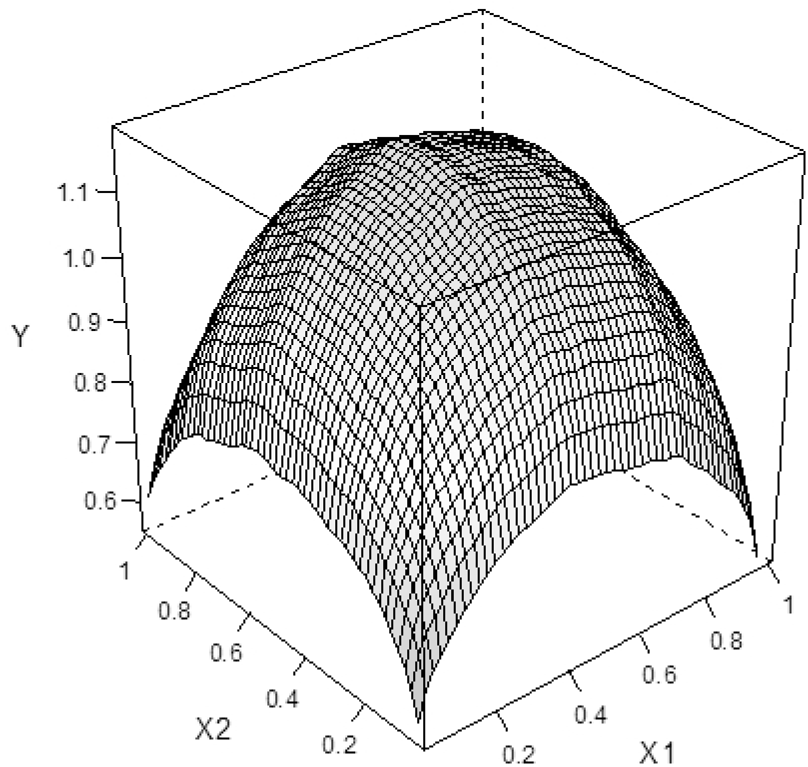

Figure 2. Density function $f_0(x1, x2)$.

of the first triangle. For the reasons of cost and ease of the implementation of the research, it is important to choose these 3 elements relatively close to each other.

Let us assume that one has information which suggests that values of Y change in the majority of the population monotonically (precisely, weakly monotonically, which means some that of the values of Y can be party constant). Let us denote Y-values of the first 3 sampled elements by $y_{1,1}$, $y_{1,2}$, $y_{1,3}$.

Consider 3 points $(x1_{1,1}, x2_{1,1}, y_{1,1})$, $(x1_{1,2}, x2_{1,2}, y_{1,2})$, $(x1_{1,3}, x2_{1,3}, y_{1,3})$, embedded in three-dimensional space $X1 \times X2 \times Y$. Selection of the next, fourth element is dependent on the values $y_{1,1}, y_{1,2}$ and $y_{1,3}$. The condition which defines sampling of the fourth element is as follows:

$$\exists_{i,j \in \{1,2,3\}, i \neq j} \quad y_{1,i} \leq \bar{y} \wedge y_{1,j} \geq \bar{y}. \tag{11}$$

There are, of course, two possible situations (compliance and non-compliance with condition (11)):

- compliance with condition (11). At least one of the elements $y_{1,1}$, $y_{1,2}$, $y_{1,3}$ has the value equal or lower than $\bar{y}$ and at least one of the elements $y_{1,1}$, $y_{1,2}$, $y_{1,3}$ has the value equal or higher than $\bar{y}$. Then, inside the triangle with vertices $(x1_{1,1}, x2_{1,1}, y_{1,1})$, $(x1_{1,2}, x2_{1,2}, y_{1,2})$, $(x1_{1,3}, x2_{1,3}, y_{1,3})$ we can find a point $(x1_{1,4}, x2_{1,4}, y_{1,4})$ such that $y_{1,4} = \bar{y}$. More precisely, it is a plane (segment should be treated as a degenerated plane) composed with points with given characteristics. Let us introduce the coefficient $d \in (0, 1)$. This coefficient defines the probability of sampling of the fourth element (point $(x1_{1,4}, x2_{1,4}, y_{1,4})$) from the plane inside a triangle $(x1_{1,1}, x2_{1,1}, y_{1,1})$, $(x1_{1,2}, x2_{1,2}, y_{1,2})$, $(x1_{1,3}, x2_{1,3}, y_{1,3})$. Therefore, the value of d coefficient should be set after the first triangle is built. When the value of d coefficient is set, the area (relative to population) of the first triangle should be taken into account. The smaller area of the first triangle is, the lower the value of d coefficient should be. However, other factors should also be considered, such as the diversity of population. The main feature of d coefficient is that the higher the value of d is, the greater 'adaptability' (ability to learn on already sampled elements) the triangular method of spatial sampling has.

  Naturally, coefficient $1 - d$ defines the probalitity of sampling of an element outside the triangle, but located relatively "close" to the triangle. In the second case, to sample the point it is suggested to use the sampling scheme which prefers elements situated in the neighborhood of the first selected element, such as sampling method presented in Chapter . In other words, the fourth element could be sampled in the same way in which the third and the second element were sampled.
- non-compliance with condition (11). With probability 1 we select an element located relatively "close" to the triangle $(x1_{1,1}, x2_{1,1}, y_{1,1})$, $(x1_{1,2}, x2_{1,2}, y_{1,2})$, $(x1_{1,3}, x2_{1,3}, y_{1,3})$ (cf. the sampling method presented in Chapter)

Then, having selected four points on the plane $X1 \times X2$, we create new triangles by choosing for each of the sampled points two other points which are closest to them (in the sense of Euclidean distance). Triangles with the same vertices are treated as one.

Let us consider the situation when, by using the method described above, we have created k triangles from m sampled elements (points in space). Let us denote these triangles by $\Delta_{1,1}, \Delta_{1,2}, \ldots, \Delta_{1,k}$. For each triangle we verify the compliance with the following condition:

$$\exists_{i,j\in\{1,2,3\},\, i\neq j} \quad y_{1,l_i} \leq \bar{y} \wedge y_{1,l_j} \geq \bar{y}, \tag{12}$$

where $y_{1,l_1}, y_{1,l_2}, y_{1,l_3}$ are vertices of the triangle $\Delta_{1,l}, l = 1, \ldots, k$.

Let us denote the triangles fulfilling condition (12) by $\Delta_{1,p_1}, \Delta_{1,p_2}, \ldots, \Delta_{1,p_q}$, $q \leq k$. Then, the d coefficient denotes the probability of sampling the next element from interior of one of the triangles $\Delta_{1,p_1}, \Delta_{1,p_2}, \ldots, \Delta_{1,p_q}$. Especially, the probability of sampling the next element from the interior of the triangle $\Delta_{1,i}$ is

$$p_{1,m+1}(i) = d\,\frac{h(\Delta_{1,i})}{\sum_{j=1}^{q} h(\Delta_{1,p_j})}, \quad i \in \{p_1, p_2, \ldots, p_q\}, \tag{13}$$

where $h(\Delta_{1,i})$ is the area of the projection of the triangle $\Delta_{1,i}$ on the plane $X1 \times X2$. We can also simplify this scheme by taking $h(\Delta_{1,i}) = 1$, $i \in \{p_1, p_2, \ldots, p_q\}$.

After sampling the triangle we define uniform distribution on the plane inside the triangle, on which (according to conjecture) the value of the variable under study should be equal to $\bar{y}$. In the last step we sample the point from the plane according to the uniform distribution.

The sampling procedure is carried out until the sample size is equal to n_0. After sampling of n_0 elements from the area covered by the triangles we select a new element to the sample, "distant" from the previously sampled elements. Construction of the density for such sampling could be made by analogy to construction described in . The difference is that in 1 instead of the distance function the inverse of the distance function is used (the values of the function are higher for elements more distant from the hub). On the base of the $n_0 + 1$ element presented sampling scheme is repeated. In other words, we start for the second time the initial sampling of three elements - the vertices of the next first triangle. The main feature of n_0 coefficient is quite similarly to the main feature of d coefficient. However, it could be assumed that $n_0 = n$.

Generally, condition (12) adapted for sampling from k-th area (i.e. after sampling $(k-1)n_0$ elements) is as follows

$$\exists_{i,j\in\{1,2,3\},\, i\neq j} \quad y_{k,l_i} \leq \bar{y} \wedge y_{k,l_j} \geq \bar{y}. \tag{14}$$

Finally, using empirical inclusion of probabilities of Horvitz-Thompson statistic as well as variance estimators could be calculated [4]. Empirical density function and, therefore, this empirical inclusion probabilities for the triangular method of spatial sampling could be obtained using Monte Carlo method. Naturally, the density function is strictly dependent on the values of Y characteristic. In the example presented below such function is defined.

4. Example of the triangular method of spatial sampling

Let us consider a research on the forest which aim is to approximate the ability of the trees to absorb carbon dioxide. Diameter at breast height (DBH) is a standard method of expressing the diameter of a trunk of a standing tree. In continental Europe, Australia, the UK and Canada the diameter is measured at 1.3 metres above ground [2]. This characteristic is strongly correlated with the weight of a tree, thus it could be used to estimate carbon dioxide absorption [3]. In addition, DBH average in strata can be assessed by the average age of trees in strata. Therefore, if there is a possibility to sample only few trees in strata, then it is necessary to construct a sampling scheme in such a way that the trees with DBH values close to the average in strata are choosen. Then, the risk of sampling a tree with DBH value distant from average is reduced and the precision of estimation is improved.

Let us consider the sampling of points in a space (a land on which the forest grows) which are centres of circles with fixed radius, as the method of selecting a sample in this research. A point in a space for which a part of a circle is outside the forest could be sampled. In this case the radius of the circle is increased in such way that the area of the circle inside the forest is equal to the area of a 'normal' circle. Trees inside the circle will be later used to estimate the total carbon dioxide absorption. The sample is, therefore, taken from infinite (uncountable) population, but the estimation is made from finite population (trees). This method is a common approach in studying forests (cf. Fattorini et al. 2006). Because of differences between sampling elements and elements which are used in estimation, the assumption about monotonic changes of Y in the majority of the population refers to an average DBH value inside a circle. An average value of DBH is assigned to the point in $X_1 \times X_2$ space - centres of the circle (in practice, it is assigned to part of $X_1 \times X_2$ space - circles with centres close to each other can have the same contents). Sampling is conducted with replacement. The sampled trees are cut down and weighted in the next phase of the study. If the tree was only partly located in the sampled circle, the weight of the tree is multiplied by the share of the circle in the area of the trunk 1.3 metres above ground. By relying on these measurements, one can assess the amount of carbon dioxide absorbed by the forest.

The forest can be divided into strata, using the economic map of forest area. Those maps provide ready-to-use division of the forest, based on dominant species, its share in total afforestation of the area and the average age of dominant tree species. The sample is taken from infinite (uncountable) population, thus each strata is a part of two-dimensional space, and consists of infinite (uncountable) elements - points in the space.

Referring to the theory, if the population can be divided into strata, then the division should maximize the differentiation between average values of the variable under study in strata (in other words, the aim is to maximize the intergroup variance). Thus, having strata which strongly differentiate capability to absorb the carbon dioxide (species and age are strongly correlated with this ability), it is important to select a "good" representative of each strata. Since the costs of weighting trees are very large, the perspective of sampling only a few elements in strata is very important.

Measurements of a tree diameter at breast height (DBH) on a simulated forest were used as a testing field of the triangular method of spatial sampling. In order to test this sampling method apprioprate simulations were performed. Also an apprioprate program was written in the R package. Simulations were conducted on the simulated matrix of DBH values. This matrix was equivalent to the real forest, the cells of matrix corresponded to the fragments of the forest and the values in the cells are mean DBH values in certain fragments of the forest. Matrix of DBH values was a square matrix, consisted of 10 000 cells. As a result of the simulation, a density function which determines the probabilities of inclusion of $X1 \times X2$ space fragments was obtained.

The triangular method of spatial sampling is based on selection of points from space - from an uncountable population. Therefore, the matrix of DBH values was equated to a space $[0,1] \times [0,1]$. The space $[0,1] \times [0,1]$ is a square forest area for which we can set two-dimensional coordinates. Each cell of DBH matrix is equivalent to the fragment of the space $[0,1] \times [0,1]$ of the size equal to $[0,0.01] \times [0,0.01]$. The value in each cell is the mean DBH value of the trees which grow on the area determined by coordinates of this cell in DBH matrix.

Further, as shown in the previous section, points are sampled from the space $[0,1] \times [0,1]$. Rather than choosing to sample circles centered in sampled points, as is described in Chapter , cells were selected from DBH matrix, within which sampled points were located. This way of proceeding was due to restrictions which result from algorithmization of mathematical models.

In the first step, the matrix of simulated average DBH values of the trees in certain forest was constructed. Spatial autoregressive model (SAR) was used in simulation (cf. Anselin [1980]). The form of this model was relatively simple, which facilitates further interpretation of the results of the simulation. The DBH value in the cell was influenced by the values of the adjacent cells which already have set the DBH values. In other words, cells $[i-1,j]$, $[i,j-1]$ and $[i-1,j-1]$ influenced on cell $[i,j]$, $i \in \{2,\ldots,100\}$, with following weights: 0.4, 0.4 and 0.2. For simulation of the DBH value in first the row/column of matrix, DBH value from the previous cell in the same row/column was used. The model was expanded by including a random element, which changed the value obtained from the basic model by 1% at the most. For the starting point (cell[1,1]) DBH value equal to 20cm was set. Formally, this model can be described as follows:

$$[i,j] = \begin{cases} (0.4\left([i-1,j]+[i,j-1]\right)+0.2[i-1,j-1])\left(1+0.01\varepsilon_{i,j}\right), \\ \text{ when } i,\ j \in \{2,\ldots,100\}, \\ [i-1,j]\left(1+0.01\varepsilon_{i,j}\right), \text{ when } i \in \{2,\ldots,100\},\ j=1, \\ [i,j-1]\left(1+0.01\varepsilon_{i,j}\right), \text{ when } i=1,\ j \in \{2,\ldots,100\}, \\ 20 \text{ cm, when } i=j=1, \end{cases} \tag{15}$$

where $\varepsilon_{i,j}$ $i,\ j \in \{1,\ldots,100\}$, have uniform distribution on segment $[-1,1]$.

In addition, limits on the maximum and minimum value of DBH average were imposed on the model. The average of diameter at breast height was no more than 30 cm and no less than 10 cm. Figure 3 shows the result of simulation of DBH values.

An average of DBH values on the forest created by simulation was 19.835 cm, with a standard devation equal to 0.746 cm.

On such a simulated forest the triangular method of spatial sampling was conducted. The sample consisted of 20 elements. At the first stage (initial sampling) three points from the space $[0,1] \times [0,1]$ were sampled and used to create a triangle. Further, 17 elements were sampled either from interior of one of the triangles, in compliance with condition (14) (with probability $d = 0.9$), or in accordance with density defined after drawing the first element (with probability $1 - d = 0.1$). Probabilities of sampling each of the triangles are the same, that is they are not weighted by the areas of projection of triangles on space $X1 \times X2$. As the expected average DBH value (the constant $\bar{y}$) 20 cm was set.

Sampling of 20 elements was repeated 10 000 times. The necessary time to make all simulations was nearly 6 hours. 200 000 observations were obtained this way. Using Monte Carlo method the density function was obtained. This density function determines the probability of sampling of each part of the forest. This density function is, of course, strongly dependent on the average DBH value obtained by simulation. Density function is shown in Figure 4.

The average DBH from 200 000 sampled elements was 19.966 cm, with the standard devation equal to 0.593 cm. As expected, the sampled elements were close to the sought value, which was 20 cm. In addition, standard devation from elements in sample was less than standard deviation in the population.

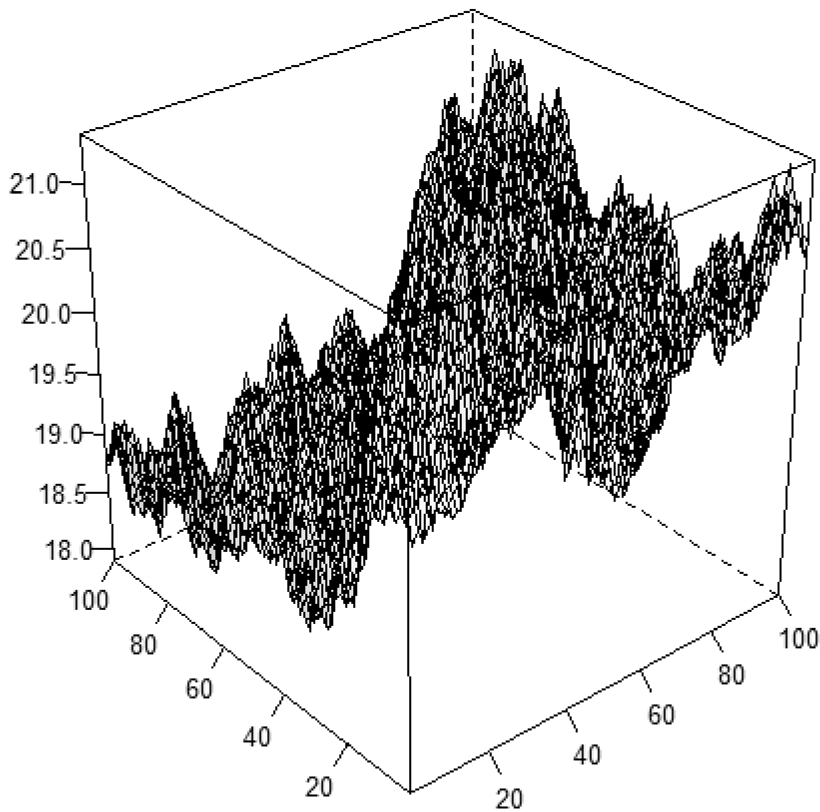

Figure 3. Simulated diameter at breast height.

5. Conclusion

The triangular method of spatial sampling can be an interesting alternative to classical methods of spatial sampling, especially in the case of stratified sampling, where this method allows the selected elements around a predetermined value to e stabilized. As a result, triangular method of spatial sampling increases the chance of achieving a high interstrata variance, which is a desirable feature in proportional stratified sampling.

It should be emphasized that an important advantage of the triangular method of spatial sampling could be the reduction of the cost of research. For instance, if $[0, 1] \times [0, 1]$ is a space which defines location of the element (for example length and width), then by choosing elements which are close to

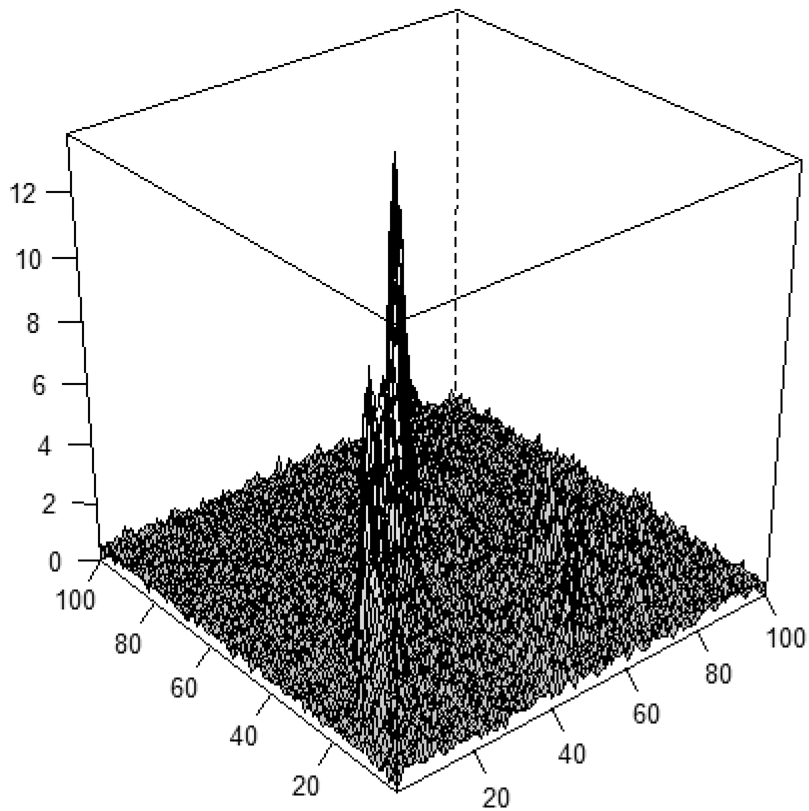

Figure 4. Density function obtained by Monte Carlo method.

each other, time and costs of research could be reduced. This aspect is particularly important in the case of spatial sampling, where the implementation is often a very expensive aspect of survey. It should be noted that for the efficiency of implementation of the triangular method of spatial sampling mobile electronic devices are necessary. One should not, however, consider this as a serious problem. Statistics is a branch of science more and more computerized, so one should try to begin using computers (especially mobile electronic devices) in the process of the research implementation. Then, a response to elements included in the sample could be made already during the implementation of the research. The response to elements included in the sample can be done by changing the probabilities of inclusion or transferring to/focusing on a certain fragment of the population.

In the end it should be emphasized that the presented method can be the subject of modifications, depending on the needs of the researcher.

The author thinks that the change of $\bar{y}$ – constant value into a variable could be an interesting problem, which gives new posibilities to use the triangular method of spatial sampling.

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